



MARKSCHEME

May 2005

MATHEMATICAL STUDIES

Standard Level

Paper 2

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Paper 2 Markscheme

Instructions to Examiners

1 Method of marking

- (a) All marking must be done using a **red** pen.
- (b) Marks should be noted on candidates' scripts as in the markscheme:
- show the breakdown of individual marks using the abbreviations *(M1), (A2) etc.*
 - write down each part mark total, indicated on the markscheme (for example, *[3 marks]*) – it is suggested that this be written at the end of each part, and underlined;
 - write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- penalise an error when it **first occurs**;
- accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- award **M** marks for a correct method, and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate’s Script	Marking	
$\$ 600 \times 1.02$	<i>MI</i>	Amount earned = $\$ 600 \times 1.02$	✓	<i>MI</i>
= $\$ 612$	<i>AI</i>	= $\$602$	×	<i>A0</i>
$\$ (306 \times 1.02) + (306 \times 1.04)$	<i>MI</i>	Amount = $301 \times 1.02 + 301 \times 1.04$	✓	<i>MI</i>
= $\$ 630.36$	<i>AI</i>	= $\$ 620.06$	✓	<i>AI(ft)</i>

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate’s working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by “**(d)**” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by “**(d)**” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc.* Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \qquad \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect, i.e. once the correct answer is seen, ignore further working.

- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , $1\cdot7$, $1,7$; different forms of vector notation such as \vec{u} , \overline{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error (**AP**).

Award the marks as usual then write $-1(\mathbf{AP})$ against the answer and also on the **front** cover

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
- OR**
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks

Throughout the mark scheme wherever G marks are awarded, these are designated for answer seen without working only. Follow through is always assumed from one question part to the next unless otherwise instructed.

QUESTION 1

(a) Profit = Income – Cost

$$P(x) = 150x - 0.6x^2 - (2600 + 0.4x^2) \quad (M1)$$

$$= 150x - 0.6x^2 - 2600 - 0.4x^2 \quad (M1)$$

Note: Award (M2) for either line seen without the other, but award only (M1) if omission of brackets results in $+0.4x^2$.

$$= -x^2 + 150x - 2600 \quad (AG)$$

[2 marks]

(b) maximum profit when $x = -\frac{-150}{2 \times -1}$ or $x = -\frac{20 + 130}{2}$ (M1)

$$= 75 \text{ machines} \quad (A1)$$

or (G2)

Note: Sketch or table of values from GDC can receive (M1) as long as the values are appropriate. Table must include at least evaluation for 74, 75, 76, and sketch must show 75 beneath the maximum, however, any non-integer answer must receive (A0). If differentiation is used, award (M1) for $-2x + 150 = 0$.

(c) $I(75) = 150(75) - 0.6(75)^2$ [2 marks] (M1)

$$= \$ 7875 \quad (A1)$$

$$\text{Selling price per machine} = \frac{7875}{75} \quad (M1)$$

$$= \$ 105 \quad (A1)$$

Notes: If P(75) or C(75) used, award M0A0 but ft with candidate's value to the selling price.

(d) $P(x) = 0$ or $(x - 20)(130 - x) = 0$. [4 marks] (M1)

$$x = 20 \text{ (130 need not be mentioned)} \quad (A2)$$

Smallest number must be 21. (A1)

Notes: If no working shown:
Award (G2) if answer is 20, $x > 20$ or $x = 20$,
Award (G3) if answer is 21 or $x = 21$.
A sketch of the function showing the intercepts receives (M1) with (A2) or (A3) for separate indication of answer 20 or 21 respectively.
If brackets are expanded and quadratic formula is used, the (M1) should be awarded only for correct expansion and correct substitution into the formula.

[4 marks]

Total [12 marks]

QUESTION 2

Note on use of radians in question 2:

In (i)(a) the answer will be -874. Award **(A0)** at the last step for either + or - 874.
 In (b) follow through with either sign from (a) can receive **(M1)(A1)ft**, but in both cases the final answer of ± 947000 receives **(A0)** for unrealistic sign and/or for unrealistic magnitude.
 In (ii) the height now becomes 13.0609... (though this value is not required).
 The volume will be 298.
 If marks were lost for radian use earlier, then 298 can receive full **ft** marks, but note that 299 is the result of premature rounding of the height to 13.1 above and so must be awarded **(A0)**.
 If marks were **not** lost in part (i) due to radian use (e.g. question not completed), then the first three marks can be awarded for working but the answer of 298 receives **(A0)**.

(i) (a) Third angle of triangle = $180 - (75 + 40)$ **(M1)**
 $= 65^\circ$ **(A1)**

Notes: Award **(A2)** for 65 seen.
 For use of 40° or 75° in an otherwise correct sine rule award **(M1)(A0)(A0)**

Length of fence: $\frac{x}{\sin 65^\circ} = \frac{410}{\sin 75^\circ}$ (sine rule) **(M1)(A1)**
 $x = 385 \text{ m (3 s.f.)}$ **(A1)**
or (G2)
[5 marks]

(b) Area = $\frac{1}{2}ab \sin c$
 area = $\frac{1}{2} \times 385 \times 245 \sin 24^\circ$ **(M1)(A1)**
 $= 19\,200 \text{ (m}^2\text{) (3 s.f.)}$ **(A1)**
or (G2)
[3 marks]

(ii) $\frac{\text{height}}{5.7} = \tan 42^\circ$, **(M1)**
 therefore height = $5.7 \tan 42^\circ (= 5.1323\dots\text{cm})$ **(A1)**
or (G2)
 Volume of prism = $\frac{5.7 \tan(42^\circ) \times 5.7 \times 8}{2}$ **(M1)**
 $= 117 \text{ cm}^3 \text{ (3 s.f.)}$ **(A1)**
or (G2)

Note: The only departures from the substituted volume formula allowed are those where the $5.7 \tan(42)$ is replaced with a value that the candidate seems to believe is the height. e.g. 5.7 repeated is a possibility. In such cases, award **(M1)(A0)**.

[4 marks]

Total [12 marks]

QUESTION 3

- (i) (a) (i) $q \Rightarrow p$ (AI)(AI)
 (ii) $\neg p \Rightarrow \neg q$ (AI)(AI)

Note: In both (i) and (ii) award (AI) for both correct statements and an implication sign present then (AI) for implication in the correct direction (can be to left or to right as long as statements are positioned correctly relative to the implication).
 In (ii), if the connective is the same incorrect connective as used in (i) but both the negations are present, then award (AI)(A0).
 Note also that a candidate with exceptional knowledge might write $p \vee \neg q$ for (i) and $\neg q \vee p$ for (ii). These are both logically equivalent to the answers and should be marked correct, however, any variations of these follow the scheme described before.

[4 marks]

- (b) If I walk to school then the sun is shining. (AI)(AI)

Note: Award (AI) for ‘if I (will) walk to school’ and (AI) for both statements in the correct order. ‘Then’ can be omitted. As in part (i) there is a logically equivalent version ‘I do not walk to school or the sun is shining’ which should be marked correct if seen.

[2 marks]

- (ii) (a)
- | p | q | $p \wedge q$ | $p \vee q$ | $\neg p$ | $(p \vee q) \wedge \neg p$ | \Rightarrow | q |
|-----|-----|--------------|------------|----------|----------------------------|---------------|-----|
| T | T | T | T | F | F | T | T |
| T | F | F | T | F | F | T | F |
| F | T | F | T | T | T | T | T |
| F | F | F | F | T | F | T | F |
- (AI)(AI)(AI)

Note: A correct column **always** receives (AI). If a column is incorrect, **ft** at each subsequent step.
 Whole table does not need to be redrawn, just the 3 columns is enough.

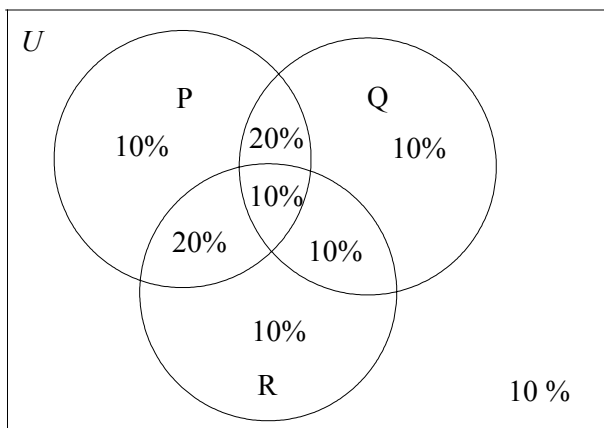
[3 marks]

- (b) Valid argument or tautology (do not accept “true”) (AI)

Note: Follow through from candidate’s table as follows:
 If the implication column is empty award A1 for tautology otherwise **ft** must be appropriate: invalid or not valid for mixed column or contradiction for four F’s.

[1 mark]

(iii) (a)



(A4)

Notes: Award **(A1)** for labelled sets P, Q, and R included inside a universal set. (Label U is not essential.)
 Award **(A1)** for central entry 10%, **(A1)** for 20%,20%,10% in the other intersecting regions,
 Award **(A1)** for the remaining three 10% in P,Q and R.
ft at each **stage for numerical errors**, however, 10 followed by 30,30,20 then 60,50,50 receives only **(A1)** for the initial 10.
 Allow fraction or decimal notation or missing % sign.
 The 10% outside of P, Q, and R can be omitted.

[4 marks]

Note: For (b) to (d) **ft** from the candidate's diagram, but not if the answer is negative or (strictly) greater than 100%

(b) 50 % read exactly two magazines

(A1)

[1 mark]

(c) 60 % read at least two magazines

(A1)

[1 mark]

(d) 10 % do not read any magazines

(A1)

[1 mark]

Total [17 marks]

QUESTION 4

Accept all answers given as percentages. If a correct fraction is seen as answer, ignore subsequent cancellations or decimal reductions (including AP's) if incorrect.

Note In each of (a)(i), (a)(iii), (b)(i) and (b)(ii) award **(AI)** for numerator and **(AI)** for denominator. **Ft** if the denominator is incorrect.

(a) (i) $P(\text{shady}) = \frac{32}{60} \left(= \frac{8}{15} \text{ or } 0.533 \right)$ **(AI)(AI)**

Note: Award **(AI)** for numerator, **(AI)** for denominator.

(ii) $P(\text{dark and low growth rate}) = \frac{8}{60} \left(= \frac{2}{15} \text{ or } 0.133 \right)$ **(AI)**

(iii) $P(\text{not dark}) = \frac{60-11}{60} = \frac{49}{60} (= 0.817)$ **(AI)(AI)**

[5 marks]

(b) (i) $P(\text{high growth rate or dark environment})$
 $= \frac{11+25-6}{60} \left(\text{or } \frac{8+14+8}{60} \right) = \frac{30}{60} \left(= \frac{1}{2} \text{ or } 0.5 \right)$ **(AI)(AI)**

(ii) $P(\text{in light, given high growth rate}) = \frac{8}{25} (= 0.32)$ **(AI)(AI)**

[4 marks]

(c) (i) $P(\text{CC}) = \frac{8}{60} \times \frac{7}{59} = \frac{56}{3540} \left(= \frac{14}{885} = 0.0158 \right)$ **(M1)(AI)**
or (G2)

Note: Award **(M1)** for multiplication of two fractions, **(AI)** for the answer.

(ii) Three alternative valid approaches might be seen:

$$\frac{23}{60} \times \frac{22}{59}$$

OR $\frac{15}{60} \times \frac{14}{59} + \frac{15}{60} \times \frac{8}{59} + \frac{8}{60} \times \frac{15}{59} + \frac{8}{60} \times \frac{7}{59}$

OR $1 - \left(\frac{37}{60} \times \frac{36}{59} + \frac{37}{60} \times \frac{15}{59} + \frac{15}{60} \times \frac{37}{59} + \frac{37}{60} \times \frac{8}{59} + \frac{8}{60} \times \frac{37}{59} \right)$ **(M1)(AI)**

$= \frac{506}{3540} \left(\text{or } \frac{253}{1770} \text{ or } 0.143 \right).$ **(AI)**
or (G3)

Note: If $= \frac{23}{60} \times \frac{22}{59}$ is doubled, award **(M1)(A0)(A0)**.
 If non-replacement is treated incorrectly in (i), **ft** to (ii) if consistent

[5 marks]

Total [14 marks]

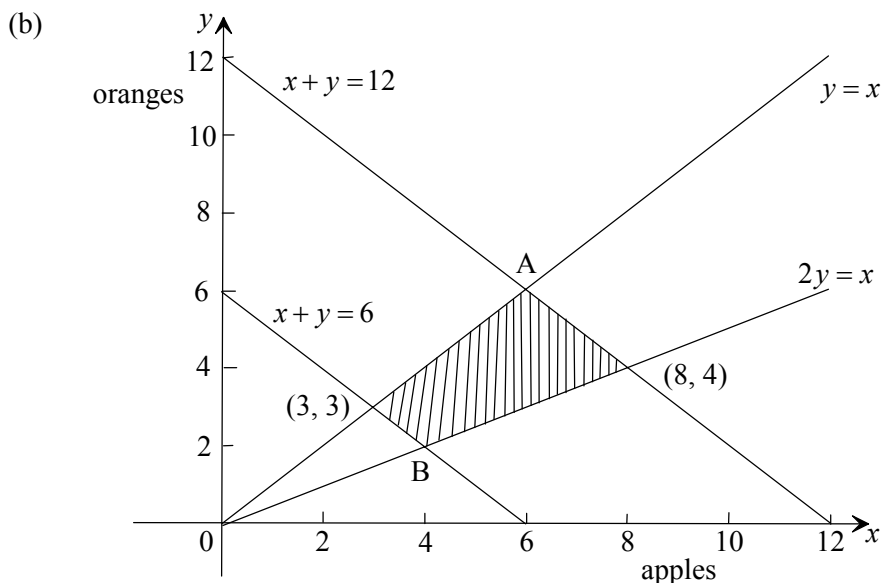
QUESTION 5

- (a) $x + y \geq 6$ (A1)
 $x + y \leq 12$ (A1)
 $2y \geq x$ or equivalent. (A1)

Note: If strict inequalities are given **consistently**, then can **ft** after awarding (A0) for the first. No mark should be awarded for any equality.

If inequalities appear only on the figure, these can be regarded as answers for both (a) and (b) and marked according to the instructions for those parts.

[3 marks]



- (i) Award one mark for each missing equation positioned next to the corresponding line. (A3)

Note: If an **inequality** is written next to the correct line, penalize the first time then allow **ft**.
 If the original inequalities are wrong, award marks according to whether there is any apparent reasoning in the assignment of equation to line.
 For example, a wrong inequality $y \geq \frac{x}{3}$ leading to $3y = x$ attached to the line $2y = x$ could receive a **ft** mark.
 In general no mark should be awarded if an equation with non-zero axis intersections is assigned to a line passing through the origin, or vice versa.
 Award (A0)(A0) if $x + y = 6$ and $x + y = 12$ are reversed.

- (ii) For A: solve $y = x$ and $x + y = 12$

$2x = 12$ (M1)
 A is (6, 6) (A1)
 or (G2)

continued...

Question 5 continued

For B: solve $2y = x, x + y = 6$

$$3y = 6 \text{ or } \frac{3x}{2} = 6$$

B is (4, 2)

(M1)

(A1)

or (G2)

Note: In both cases, both the values must be correct to receive the A1 or G2

[7 marks]

(c) (i) Candidates should indicate that they tested at least the following corners:

(3, 3)	$6 + 3 =$	9
(4, 2)	$8 + 2 =$	10

(M1)

Both correct

(A1)

Choose 3 of each.

(A1)

or (G2)

Note: Wrong choice without working (probably (4,2)) receives no marks but can follow through to (ii).

Minimize cost to maximize change (or equivalent).

(R1)

(ii) Cedric keeps \$ 2.

(A1)

[5 marks]

Total [15 marks]

QUESTION 6

(i) (a) $A^T = \begin{pmatrix} 2 & c \\ -3 & 3 \end{pmatrix}$ (A1)

[1 mark]

(b) (i) $\det(A) = 2 \times 3 - (-3) \times c$ (M1)
 $= 6 + 3c$ (A1)

Note: Award (A2) for correct answer with no working.

(ii) Singular A requires $6 + 3c = 0$, (M1)
hence $c = -2$. (A1)

[4 marks]

(c) $A^2 = \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -15 \\ 5 & 6 \end{pmatrix}$ (A1)(M1)(A1)

Note: Award (A1) for substituting $c = 1$, (M1) for multiplying, (A1) for correct answer.

OR $A^2 = \begin{pmatrix} 1 & -15 \\ 5 & 6 \end{pmatrix}$ (G3)

$$A^2 - 5A = \begin{pmatrix} 1 & -15 \\ 5 & 6 \end{pmatrix} - 5 \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix}$$

OR $= \begin{pmatrix} 1 & -15 \\ 5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -15 \\ 5 & 15 \end{pmatrix}$ (M2)

$$= \begin{pmatrix} -9 & 0 \\ 0 & -9 \end{pmatrix}$$
 (A1)
or (G1)

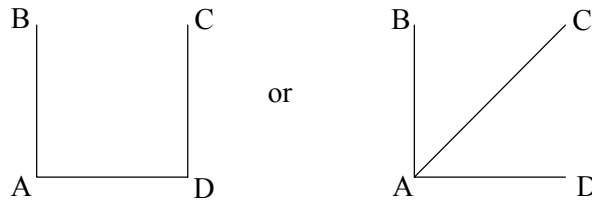
When $c = 1$, $\det(A) = 6 + 3c = 9$, so left hand side $= -\det(A) \times I$. (A1)(AG)

[7 marks]

continued...

Question 6 continued

(ii) (a) (i) For example

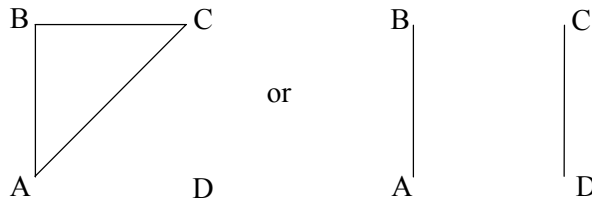


or any other properly labelled graph which is connected.

(A1)(A1)

Note: Award (A1) for four vertex connected graph, (A1) for labelling.

(ii) For example



or any other properly labelled graph which is disconnected.

(A1)(A1)

Note: Award (A1) for four vertex disconnected graph, (A1) for labelling.)

[4 marks]

- (b) (i) $d(C) = 3$ (A1)
- (ii) $x = 0, y = 1$ (A1)(A1)
- (iii) $A_{E \rightarrow G}^2 = 3$ (A1)
- (iv) **EITHER** from direct counting $k = 3$ (A2)

OR from matrix multiplication $M_{7,7}^2 =$

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 1+1+1=3 \quad (M1)(A1)$$

[6 marks]

continued...

Question 6 continued

Note: In all cases below, winnings are asked for and losses of the other player receive *(A0)*. Follow through from (b) to (c) and (d).

- (iii) (a) Wins 2 or +2 or 2. *(A1)*
[1 mark]

 - (b) Yuichi chooses wolf, Katerina chooses lion. *(A1)(A1)*
[2 marks]

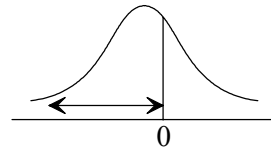
 - (c) (i) Elephant. *(A1)*
(ii) Wins 1 or +1 or 1. *(A1)*
[2 marks]

 - (d) (i) Mouse. *(A1)*
(ii) Wins 3 or +3 or 3. *(A1)*
[2 marks]

 - (e) Unstable. *(A1)*
[1 mark]
- Total [30 marks]***

QUESTION 7

(i) $P\left(z \leq \frac{0+5}{9} = 0.5556\right)$ or $P(x < 0, N(-5, 9))$ etc or



(M1)

= 0.711 or 0.712

(A1)

or (G2)

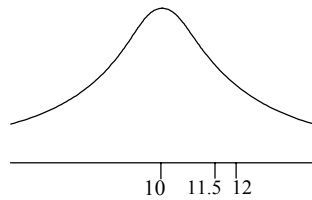
[2 marks]

(ii) (a) 50 %

(A1)

[1 mark]

(b) (i)



bell-shaped curve

(A1)

all positions clearly identified (must be 11.5, not 8.5)

(A1)

(ii) Region clearly identified, (shading to right of 12).

(A1)

(iii) $z_1 = \frac{11.5 - 10}{1.5} = 1.0$ $z_2 = \frac{12 - 10}{1.5} = 1.3\bar{3}$

So evaluate $P(1.0 < z < 1.3\bar{3})$

OR evaluate $P(11.5 < x < 12, N(10, 1.5))$

OR correctly shaded bell-shaped curve

OR 0.0674(4) seen (allow 0.0669 from use of 1.33)

(M1)

Hence number of days = 0.0674×200 (or 0.0669×200)

(M1)

= 13 days (accept 13.4 or 13.5)

(A1)

or (G2)

[6 marks]

(c) $x = (-1.645)(1.5) + 10 = 7.53$ minutes

(M1)(A1)

or (G2)

Note: Award (M1)(A0) for using +1.645.
Award (G2) for 7.54 seen without working.

[2 marks]

continued...

Question 7 continued

- (iii) (a) H_0 : There is no significant difference between the data and a normal distribution with $\mu = 9, \sigma = 4$. (A1)

Note: Accept equivalent less formal statements including “data is normally distributed”, “data fits whole industry” or just $\mu = 9, \sigma = 4$.
Mention of μ and σ is not essential.

H_1 : ft from H_0 e.g. $\mu \neq 9, \sigma \neq 4$. (A1)

[2 marks]

- (b) $p = 33.4$ (A1)

[1 mark]

- (c)

$f_e - f_o$	$\frac{(f_e - f_o)^2}{f_e}$
5.9	2.189
-4.0	0.364
-5.6	0.939
3.7	2.043

(A1)(A1)

$\chi^2_{\text{calc}} = 5.535$ (or 5.54) (A1)
or (G3)

[3 marks]

- (d) Degrees of freedom $(n - 1) = 4 - 1 = 3$ (A1)

[1 mark]

- (e) $\chi^2_{\text{crit}, 0.05} = 7.815$, (A1)

$\chi^2_{\text{calc}} < \chi^2_{\text{crit}}$ (M1)

The data can be modelled by a normal distribution with mean of 9 % and standard deviation of 4 %... or do not reject H_0 (allow accept H_0) (R1)

Note: The conclusion must follow correctly from the candidate’s value of χ^2 to receive (R1).

[3 marks]

continued...

Question 7 continued

(iv) (a) $r = -\frac{3.77}{1.94 \times 2.35} = -0.827$

(M1)(A1)
or (G2)

[2 marks]

(b) moderate/strong (allow approximately linear)
negative

(A1)
(A1)

Note: Comments such as: number of sessions increases as time decreases can be awarded (A1)(A0) but “inversely proportional” receives no marks.

[2 marks]

(c) $\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = -3.77 \times 9 = -33.9$

(A1)

[1 mark]

(d) (i) $(y - 9) = -\frac{3.77}{1.94^2}(x - 6)$
 $y = -x + 15$

(M1)
(A1)
or (G2)

(ii) $-3 + 15 = 12$ seconds

(M1)(A1)
or (G2)

[4 marks]

Total [30 marks]

QUESTION 8

(i) (a) Substitute $x=1$ into $f(x)$, $f(1)=3$.

(M1)(A1)
or (G2)

[2 marks]

(b) $f'(x) = 6x^2 - 10x + 7$

(A1)(A1)(A1)

Note: If the -1 is left in and written separately then the constant is wrong so max possible is (A2).

[3 marks]

(c) Substitute $x=2$ into (b) $f'(2)=11$.

(M1)(A1)
or (G2)

Note: No ft here if original $f(x)$ is just written as answer for (b).

[2 marks]

(d) Increasing.

(A1)

[1 mark]

(e) (i) No.

(A1)

(ii) Because the gradient at $x=2$ is wrong (or wrong sign) or **any other valid reason** (e.g. $f(x)$ has an inflection not a max/min), (but note that $f(1)$ and $f(0)$ both agree, and both the formula and the graph have a single real root near to 0, so none of these are valid reasons).
A sketch of the graph from the GDC with no detailed reason can be awarded (GI) if it is reasonable.

(R1)

[2 marks]

(ii) Many candidates will present answers for (i) to (iii) in the wrong order or within the wrong question part or duplicated. Award marks for answers seen regardless of the precise organisation within parts.

(a) (i) $2x^2 + 4xh + 2h^2$
and $-3x - 3h$.

(A1)

(A1)

(ii) Simplified numerator is $4xh + 2h^2 - 3h$.

(A1)

(iii) $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h}$
 $= \lim_{h \rightarrow 0} (4x + 2h - 3)$
 $= 4x - 3$

(A1)

(A1)

Note: In part (iii) answer without working receives final (A1) only.
Full limit notation is not essential

[5 marks]

Question 8 continued

(b) Directly from the limit expression or by using antidifferentiation:

$$y(x) = 2x^2 - 3x + c$$

(ft from a(iii) allow any value of c including 0 or just an undetermined c)

(A1)(A1)

[2 marks]

Note: In part (iii) correct units are not required for the marks to be awarded.

(iii) (a) Put $t = 0$ in $h(t)$.

$$h(0) = 3 \text{ m}$$

(M1)

(A1)

or (G2)

[2 marks]

(b) (i) $v(t) = \frac{dh}{dt}$

$$= 16 - 9.8t$$

(M1)

(A1)

Note: Award (A2) for answer only.

(ii) $v(0) = 16 \text{ ms}^{-1}$

(M1)(A1)

[4 marks]

(c) (i) $16 - 9.8t = 0$ (or $v(t) = 0$)

$$t = \frac{16}{9.8} = 1.63\text{s} \quad \text{or} \quad 16 - 9.8 \times 1.63 = 0$$

(M1)

(M1)(AG)

Note: If only $v(1.63) = 0$ is seen, award only (M1)(M0) but can allow (G2) for a labelled sketch showing the maximum at approximately $t = 1.63$.

$$(ii) \quad h\left(\frac{16}{9.8}\right) = 3 + \frac{16^2}{9.8} - 4.9\left(\frac{16}{9.8}\right)^2$$

$$= 16.1 \text{ m}$$

(M1)

(A1)

Note: Answer 16 with correct working receives (M1)(A0)AP but without working receives (M0)(A0).

[4 marks]

(d) (i) $a(t) = \frac{dv}{dt} = -9.8 \text{ ms}^{-2}$

(M1)(A1)

Note: Award (A2) for answer without working.

(ii) Downwards (allow “negative” but (A0) for “decreasing”).

(A1)

[3 marks]

Total [30 marks]